

**Disclaimer:** *These notes have not been subjected to the usual scrutiny reserved for formal publications. They may be distributed outside this class only with the permission of the Instructor.*

## 4.1 Logic

Knowledge representation is first requirement of achieving intelligence out of three requirements of Newell's and Simons's Physical Symbol System Hypothesis (PSSH). This lecture presents the knowledge representation using propositional logic, predicate logic, as well as the inference rules.

Logic is formal method for reasoning. Using logic the concepts can be translated into symbolic representation, which closely approximate the meaning of these concepts. The symbolic structures can be manipulated using programs to deduce facts to carry out the form of a automated reasoning.

The *aim* of logic are to learn principles of valid reasoning as well as to discern good reasoning and bad reasoning, identifying an invalid argument, distinguishing inductive and deductive arguments, identifying fallacies and avoiding them.

The *Objective* is to equip oneself with various tools and techniques i.e., decision procedures, for the validity of a given argument, detecting and avoiding fallacies of a given deductive or inductive argument. To summarize:

### 4.1.0.1 Argumentation Theory

It is study of how conclusions can be reached through logical reasoning; that is, whether the claims soundly based on premises or not (figure 4.1). It includes the arts and sciences of civil debate, dialogue, conversation, and persuasion. It studies rules of inference, logic, and procedural rules in both artificial and real world settings.

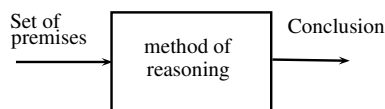


Figure 4.1: Inference Process.

Argumentation includes debate and negotiation, that are concerned with reaching mutually acceptable conclusions. It also encompasses erroneous dialogue, the branch of social debate in which victory over an opponent is the primary goal.

Argumentation is used in law, for example in trials, in preparing an argument to be presented to a court, and in testing the validity of certain kinds of evidence.

Figure 4.1 shows that an internal structure, comprising the following:

- i) a set of assumptions or premises (or antecedents)
- ii) a method of reasoning or deduction and
- iii) a conclusion or consequence.

If the premises are  $P_1, P_2, \dots, P_n$ , then they are conjuncted and their conjunction imply the conclusion  $C$ , i.e.,  $P_1 \wedge P_2 \wedge \dots \wedge P_n \rightarrow C$ .

An argument must have at least one premise and one conclusion. Often classical logic is used as the method of reasoning so that the conclusion follows logically from the assumptions or support. One challenge is that if the set of assumptions is inconsistent then anything can follow logically from inconsistency. Therefore it is common to insist that the set of assumptions be consistent. It is the practice to have minimal set.

## 4.2 Propositional Logic

The propositional logic deals with individual Propositions, which are viewed as atoms, i.e., these cannot be further broken down into smaller constituents. For building propositional logic, first we describe the logic with the help of a formula called Well-Formed Formulas (wff read as woofs). A formula is a syntactic concept, which means whether or not a string of symbols is a formula not. It can be determined solely based on its formal construction, i.e., whether it can be built according to its construction rules.

We start the propositional logic with the individual propositional variables, which are themselves are formulas hence cannot be further analyzed. We represent these by letters  $p, q, r, s, t, p_1, p_2, q_1, q_2, \dots$ , etc. These formulas may have smaller constituents but it is not the role of propositional logic to go into the details of their constructions. The use of letters to represent propositions is not in true sense variables, they simply represent the propositions or statements in a symbolic form, and they are not the variables in the sense used in predicate logic, or in high-level languages, where a variable stands for a domain of values. The other symbols of propositional logic are:

- $\wedge$  conjunction operator
- $\vee$  disjunction operator
- $\neg$  not or inverting operator
- $\rightarrow$  implication, i.e., if ... than ... rule
- $\perp$  contradiction

Following are the examples of propositions and formulas constructed using them.

$p$  = Sun is star.

$q$  = Moon is satellite.

Following formulas are constructed using above:

$p \wedge q$  = Sun is star *and* Moon is satellite.

$p \vee q$  = Sun is star *or* Moon is satellite tennis.

$\neg p \vee q =$  Sun is *not* star or Moon is satellite.

$\neg p \rightarrow q =$  *if* Moon is *not* star then Moon is satellite.

A formula in propositional logic can be defined recursively as follows:

- i) Each propositional variables is a formula therefore,  $p, q$  are formulas,
- ii) If  $p, q$  are formulas, then  $p \wedge q, p \vee q, \neg p, p \rightarrow q, (p)$ , are also formulas, and
- iii) A string of symbols is a formula only as determined by finitely many applications of above (i) and (ii).

This recursive form of the definition can be expressed by the *BNF* (Backups-Naur Form) notation as follows:

$$\begin{aligned} \text{Formula} &:= \text{atomicformula} \mid \neg \text{formula} \mid (\text{formula} \wedge \text{formula}) \\ &\mid (\text{formula} \vee \text{formula}) \mid (\text{formula} \rightarrow \text{formula}) \\ \text{atomicformula} &:= \perp \mid p \mid q \mid r \mid p_0 \mid p_1 \mid p_2 \mid \dots \end{aligned}$$

Considering two propositions  $p, q$ , the interpretation of the binary operators  $\vee, \wedge$ , and  $\rightarrow$  over  $p, q$  are shown in table 4.1.

Table 4.1: Interpretation of binary operators.

$p$	$q$	$p \vee q$	$p \wedge q$	$p \rightarrow q$
$F$	$F$	$F$	$F$	$T$
$F$	$T$	$T$	$F$	$T$
$T$	$F$	$T$	$F$	$F$
$T$	$T$	$T$	$T$	$T$

*Material conditional* joins two simpler propositions, and we write  $p \rightarrow q$ , which is read “if  $p$  then  $q$ ”. The proposition to the left of the arrow is called the *antecedent* and the proposition to the right is called the *consequent*. It expresses that  $q$  is true whenever  $p$  is true. Thus it is true in every case above except case three, because this is the only case when  $p$  is true but  $q$  is not. Using the example, if  $p$  then  $q$  expresses that if it is raining outside then there is a cold over Kashmir. The material conditional is often confused with *physical causation*. The material conditional, however, only relates two propositions by their truth-values which is not the relation of *cause* and *effect*.

## References

- [GFL09] D. GEORGE F. LUDGER, “Artificial Intelligence - Structures and Strategies for Complex Problem Solving,” *5th Edition, Pearson Education, India*, 2009, Chapter 2.
- [PATT01] DAN W. PATTERSON, “Introduction to Artificial Intelligence and Expert Systems,” *PHI India*, 2001, Chapter 4.
- [KRC12] CHOWDHARY, K.R., “Fundamentals of Discrete Mathematical Structures, 2nd Edition” *PHI India*, 2012, Chapter 5.