CS324: AI (Default logic Logic & Default Reasoning)	**Spring 2015**

Lecture 11: Feb. 16, 2015

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11.1 Default Reasoning

When decisions are based on assumptions, these may turn out to be wrong in the face of additional information that becomes available; i.e., medical tests may lead to a modified diagnosis. The phenomenon of having to take back some previous conclusions is called *non-monotonicity*; it says that if a statement φ follows from a set of premises M and $M \subseteq M'$, then φ does not necessarily follow from M' (see example 11.1). Default Logic provides formal methods to support this kind of reasoning.

Example 11.1 *Let*

$$\begin{aligned} M = \{&\forall x(bird(x) \Rightarrow fly(x)),\\ &\forall y(penguin(y) \Rightarrow bird(y)),\\ &\forall z(penguin(z) \Rightarrow \neg fly(z)),\\ &bird(tweety)\}. \end{aligned}$$

Given these, we have $M \models fly(tweety)$, and $M' = M \cup \{penguin(tweety)\}$ is inconsistent. We would expect the inference

$$M \cup \{penguin(tweety)\} \models \neg fly(tweety), \tag{11.1}$$

making $fly(tweety)$ a defeasible consequent. □

11.1.1 Notion of a Default

A rule used by football organizers in Kashmir valley might be: "A football game shall take place, unless there is snow in the stadium." This rule of thumb is represented by the default

$$\frac{football : \neg snow}{takesPlace} \tag{11.2}$$

The interpretation of the default is as follows: If there is no information that there will be snow in the stadium, it is reasonable to assume $\neg snow$ and conclude that the game will take place (so preparations can proceed). But if there is a heavy snowfall during the night before the game is scheduled, then this assumption can no longer be made. Now we have definite information that there is snow, so we cannot assume $\neg snow$, therefore the default cannot be applied. In this case we need to refrain from the previous conclusion (i.e., the game will take place), so the reasoning is nonmonotonic, as we are going to withdraw the previous conclusion.

Before proceeding with more examples, let us first explain why classical logic is not appropriate to model this situation. Of course, we could use the rule:

$$football \wedge \neg snow \rightarrow takesPlace. \tag{11.3}$$

The problem with this rule is that we have to definitively establish that there will be no snow in the stadium before applying the rule. But that would mean that no game could be scheduled in the winter, as it would not be possible to do the preparations in advance.

11.1.2 The Syntax of Default Logic

A *default theory* T is a pair (W, D) consisting of a set W of first-order predicate logic formulas (called the facts or axioms or belief set of T) and a countable set D of default rules. A default rule δ has the form

$$\delta = \frac{\varphi : \psi_1, \ldots, \psi_n}{\chi} \tag{11.4}$$

where $\varphi, \psi_1, \ldots, \psi_n, \chi$, are closed predicate logic formulae, and $n > 0$. The formula φ is called the *prerequisite*, $\psi_1, \ldots, \psi_n$ are the *justifications*, and χ the *consequent* of δ .

One point that needs some discussion is the requirement that the formulae in a default be *ground clause.*

11.1.3 Algorithm for Default Reasoning

The algorithm 1 for default reasoning provides extensions to the formula set W. Let $T = \langle W, D \rangle$ be a closed default theory (unspecified variables are false) with a finite set of default rules D and formula W. Let P be the set of all permutations of elements of default set D. If $P = \{\}$, i.e., $D = \{\}$, or if W is inconsistent, then return default theory $Th(W)$ as the only extension of T. The set of justifications and beliefs are indicated by variables $JUST$ and $BELIEF$, respectively.

Example 11.2 *Find extensions of following default theory:*

$$T = \langle W, D \rangle = \langle \{R(n) \wedge Q(n)\}, \{\frac{R(x) : \neg P(x)}{\neg P(x)}, \frac{Q(x) : P(x)}{P(x)}\} \rangle$$

Solution: First consider the permutation of D as

$$(d_1, d_2) = \{\frac{R(x) : \neg P(x)}{\neg P(x)}, \frac{Q(x) : P(x)}{P(x)}\}.$$

At begin, we initialize $BELIEF = \{R(n) \wedge Q(n)\}, JUST = \{\}$. As per algorithm 1, $d_1 = \frac{A_1 : B_1}{C_1} = \frac{R(x) : \neg P(x)}{\neg P(x)}$. From algorithm, we note that $BELIEF \vdash R(n)$, and $BELIEF \nvdash \neg(\neg P(n))$. Thus, we add C_i, i.e., $\neg P(n)$ in $BELIEF$. Also, add $\neg P(n)$ into $JUST$. We also note that, the justification does not negate the $BELIEF$, hence it is consistent.

Algorithm 1 Algorithm for Default Reasoning.

```
P = All permutations of elements of D
while P ≠ {} do
    Take a permutation, perm = {d_1, ..., d_n} ∈ P
    P = P − {perm}
    ;[initialization]
    BELIEF = W, JUST = {}
    ;[Application of defaults and consistency test]
    for i = 1 to n do
        [assume d_i = (A_i : B_i) / C_i]
        if (BELIEF ⊢ A_i) ∧ (BELIEF ⊬ ¬B_i) then
            BELIEF = BELIEF ∪ {C_i}
            JUST = JUST ∪ {B_i}
            if ∃A(A ∈ JUST and BELIEF ⊢ ¬A) then
                exit the algorithm
            end if
        end if
    end for
end while
Return Th(BELIEF)
End
```

Table 11.1: Belief Computation.

Iteration	BELIEFS	JUSTIFICATIONS	Consistency Test
Initial	$R(n) \wedge Q(n)$	$\{\}$	
$i = 1, d_1$	$\neg P(n)$	$\neg P(n)$	OK
$i = 2, d_2$	Stable		

Next, we repeat the loop of algorithm 1, for $i = 2$: $(d_2) = \frac{A_2 : B_2}{C_2} = \frac{Q(x):P(x)}{P(x)}$. Now, the updated $BELIEF = \{R(n) \wedge Q(n), \neg P(n)\}$. We note that $BELIEF \vdash A_2$ (i.e., $Q(n)$), but $BELIEF \nvdash \neg B_2$ (i.e., $\neg P(n)$) does not hold. Also, $Q(n) \in JUST$ holds, but $BELIEF \vdash \neg Q(n)$ does not hold, so algorithm continues. This conclude that belief is stable (see table 11.1).

When above is repeated for other permutation,

$$(d_2, d_1) = \{\frac{Q(x) : P(x)}{P(x)}, \frac{R(x) : \neg P(x)}{\neg P(x)}\},$$

we get the extended belief set as shown in table 11.2.

In conclusion, the T has two extensions: $Th(\{R(n) \wedge Q(n), \neg P(n)\})$, and $Th(\{R(n) \wedge Q(n), P(n)\})$. □

Table 11.2: Belief Computation.

Iteration	BELIEFS	JUSTIFICATIONS	Consistency Test
Initial	$R(n) \wedge Q(n)$	$\{\}$	
$i = 1, d_2$	$P(n)$	$P(n)$	OK
$i = 2, d_1$	Stable		