CS324: Heuristic Search methods. **Spring 2015**

Lecture 15: March 16, 2015

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15.1 Heuristic Approach

The search efficiency can improve tremendously - reducing the search space, if there is a way to order the nodes to be visited in such that most promising nodes are explored first. These approaches are called *informed* methods. These methods depend on some heuristics determined by the nature of the problem. The heuristics is defined in the form of a function, say f, which some how represents the mapping to the total distance between start node and the goal node. For any given node n, the total distance between start and goal node is $f(n)$, such that,

$$f(n) = g(n) + h(n). \tag{15.1}$$

where $g(n)$ is distance between start node and the node n, and $h(n)$ is the distance between node n and the goal node. We note that $g(n)$ can be easily determined and can be taken as shortest. The example for h is, in the 8-puzzle problem, the next move is chosen the one having minimum *disagreement* from the goal, i.e., having minimum number of misplaced positions with respect to the goal. The heuristic methods, reduce the state space to be searched, and supposed to give the solution, but my fail also.

15.2 Hill-Climbing Methods

The name hill-climbing comes from the fact that to reach to the top of a hill, one selects the steepest path at every node, out of the number of alternatives available. Naturally, one has to sort the slope values available, pick up the direction of move having highest angle, then reach to the next point (node) toward the hill top, then repeat the process. The hill-climbing algorithm 1 is an improved variant of the depth-first search method.

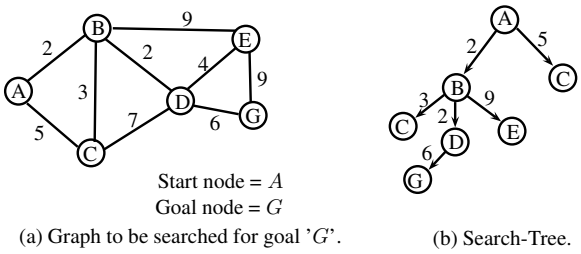
Consider the graph shown in figure 15.1(a), where start node is A and goal node is G. It is required to find out the shortest path from node A to node G, using the method of hill-climbing. Figure 15.1(b) shows the search-tree for reaching to goal node G from start node A, with shortest path A, B, D, G and path length 10. We note that this path is in fact shortest, but it can be easily worked out that this approach cannot lead to shortest always.

Hill climbing suffers from various problems:

1. Rotating the brightness knob in control panel of an analog TV does not improve the quality of picture,
2. While testing a program, running it again and again, with different data sets does not indicate new discovery of errors,

Algorithm 1 Hill-Climb(Input: **G**, **S**, **Goal**)

```
1: Open = [S]
2: Closed = nil
3: if Open = nil then
4:    return fail
5: end if
6: repeat
7:    if Open.Head = Goal then
8:       return success
9:    end if
10:   expand Open.Head and generate children's set, call it C
11:   reject all paths in C having loops
12:   delete Open.Head and insert it into Closed
13:   sort C in order of heuristic, with best heuristic node in the front
14:   insert C at the front of List
15: until Open = nil
16: Return fail
```



(a) Graph to be searched for goal 'G'. (b) Search-Tree.

Figure 15.1: Graph with Hill-climbing search.

3. Participating in a sports again and again (without new ideas and training), does not improve further performance.

Local Vs. Global Search

The preference for local search where only one state is considered at a time (out of the newly explored states), is good for a memory efficiency, compared to expanding all the successor nodes, but appear to be a too extreme step. Thus, instead, k best nodes out of successors generated are considered for further expansion. This gives rise to a new method, called *local beam search*.

However, the local beam search too will lead to concentration to a specific direction to those successors which generate more potential nodes. Consequently, this search ultimately becomes a local search. A better solution is to elect these k nodes such that they are not the best successors, but randomly selected, out of the next generation of successor nodes. This new method is called *stochastic beam search*.

15.3 Best-First Search

The heuristic approach typically uses special knowledge about the domain of the problem being represented by the graph to improve the computational efficiency of solution to particular graph-searching problem. However, the procedures developed via the heuristic approach generally have not been able to guarantee that minimum cost solution paths will always be found.

Given a weighted directional graph $G = (V, E, W)$ with a distinguished start node **S** and a set of goal nodes R, the optimal path problem is to find a least-cost path from **S** to any member of R where the cost of the path may, in general, be an arbitrary function of them, weights assigned to the nodes and branches along that path. A general best-first (GBF) strategy will pursue this problem by constructing a tree T of selected paths of G using the elementary operation of node expansion, that is, generating all successors of a given node. Starting with **S**, GBF will select for expansion that leaf node of T that features the highest "merit," and will maintain in T all previously encountered paths that still appear as viable candidates for *sprouting* an optimal solution path. Due to its nature of search, the best-first search is also called *branch-and-bound method*, i.e., branching a search to other direction to which path cost is minimum, and bounding the cost to that minimum, until a better minimum is found after next expansion.

15.3.1 GBFS Algorithm

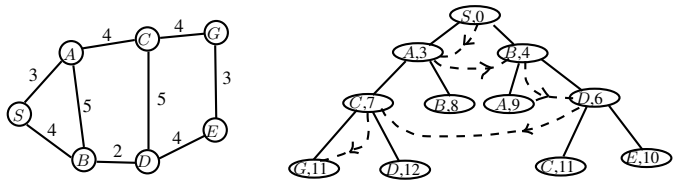
A best-first search algorithm maintains two lists of nodes, an 'Open-list'and a 'Closed-list'. The Closed-list contains those nodes that have been expanded, by generating all their children, and the Open-list contains those nodes that have been generated, but not yet expanded.

Since, best node is selected every time, it is guaranteed to give the best solution. The value of heuristic function $f(n)$ for a given node n, does not here include the the distance from current node to the goal node, as required in equation (15.1, page no. 15-1), however, since the best path is chosen every time, it is likely to provide the optimum solution for the problem.

The algorithm 2 shows the steps for best-first search.

Example 15.1 *Best-first search.*

Figure 15.2(a) shows the graph, and figure 15.2(b) shows the search tree using GBFS for reaching to goal node G. The order in which the nodes are explored is shown with dotted line. Since G is goal, its path from root is S, A, C, G with shortest path length 11. Note that any other path will be of longer or equal length.



(a) Graph

(b) Best-first Search

Figure 15.2: Best-First (Branch-and-Bound) Search.

□

Algorithm 2 Best-first Search(Input: **S**, **Goal**)

```
1: Open = [S]
2: Closed = []
3: repeat
4:    if Open.Head = Goal then
5:       return success
6:    end if
7:    generate children's set C of Open.Head
8:    if n ∈ C already exists in OPEN and new n is reachable by shorter path then
9:       remove the old n
10:   end if
11:   if n ∈ C already exists in Closed and reachable by shorter path then
12:      replace n ∈ C by the same node from Closed, along with shorter distance from root
13:   end if
14:   remove Open.Head and insert into Closed
15:   update distance from root for all C nodes
16:   add all C to either side of Open and record their parents
17:   sort Open by path length so that least cost path node is at front
18: until Open = nil
19: return fail
```

15.3.2 Analysis of Best-first search

If the evaluation function $f(n)$ used for node selection always provides optimistic estimates of the final costs of the candidate paths evaluated, then we can terminate the search as soon as the first goal node is selected for expansion, without compromising the optimality of the solution used. This guarantee is called *admissibility*.

Second, we are often able to purge from tree T, large sets of paths that are recognized at an early stage to be dominated (i.e., superior) by other paths in T. This becomes particularly easy if the evaluation function f is *order preserving*, that is, if, for any two paths p_1 and p_2, leading from **S** to n, and for any common extension p_3 of those paths, the following holds (figure 15.3):

$$f(p_1) \geq f(p_2) \Rightarrow f(p_1 p_3) \geq f(p_2 p_3) \tag{15.2}$$

It simply states that, if p_1, is judged to be more meritorious than p_2, both going from **S** to n, then no common extension of p_1 and p_2 may later reverse this judgment. Under such conditions, there is no need to keep in T multiple copies of nodes. Each time the expansion process generates a node n that already resides in T, we maintain only the lower-path to it, discarding the link from the more expensive father of n. This has been illustrated in figure 15.3.

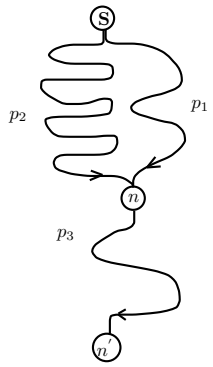


Figure 15.3: Order Preserving in GBFS.

If $f(n)$ is the total depth of node n (not the distance from start), best-first search becomes breadth-first search. Note that breadth-first searches all the closer nodes (to start) before farther nodes. If $f(n) = g(n)$, where $g(n)$ is the cost of the current path from the start state to node

n, then best-first search becomes Dijkstra's single-source shortest-path algorithm. If $f(n) = g(n) + h(n)$, where $h(n)$ is a heuristic estimate of the cost of reaching a goal from node n, then best-first search becomes a new algorithm A^* algorithm.

Breadth-first search can terminate as soon as a goal node is generated, while Dijkstra's algorithm and A^* must wait until a goal node is chosen for expansion to guarantee *optimality.*

15.3.3 Search Algorithm A^*

We can define the admissible search algorithm 3 for A^* (A-star).

Algorithm 3 Admissible-search A^*(Input: **G**, **S**, **Goal**)

```
1: Open = [S]
2: Closed = []
3: repeat
4:    compute f*(S)
5:    Select the open node n whose f*(n) is smallest
6:    Resolve ties arbitrarily, but always in favor of any node n ∈ Goal
7:    if n ∈ Goal then
8:       move n to Closed
9:       terminate algorithm
10:   else
11:      insert n to Closed
12:      apply successor operator to n
13:      calculate f* for each successor n_i of n
14:      insert all n_i ∉ Closed to Open
15:      move to Open any n_i in Closed and for which f*(n_i) is smaller now than it was when n_i was in
         Closed
16:   end if
17: until Open = nil
18: return fail
```

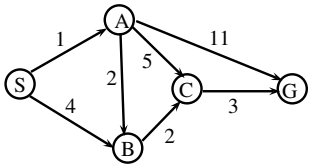
Example 15.2 *A^*-Search.*

The figure 15.4 shows a graph, heuristic function table for $h(n)$ for every node in the graph, and tree constructed for A^* search for the graph, for given start state S and goal state G. To expand the next, the one having smallest value of function f is chosen out of the nodes in the frontiers. The function f for a node n is sum of three values: the g value of the parent of n, the distance from parent of n to the node n, and heuristic value (estimated distance from n to goal, given in the table) indicated by h. In the case of a tie, i.e., two states having equal values of f, the one to the left of the tree is chosen.

Based on the criteria set for A^* (i.e., to always expand the node having smallest value of f), the order of nodes expanded for figure 15.4(a), and shown in search-tree in figure 15.4(c) with start node S and goal node G are: $(G, 0)$, $(B, 5)$, $(A, 7)$, $(B, 5)$, $(C, 6)$,$(C, 7)$ (with parent A), $(C, 7)$ (with parent B), $(G, 8)$, $(G, 9)$, $(G, 12)$. Finally, we note that the best path is corresponding to goal $(G, 8)$, and it is: S, A, B, C, G. Note that, in the A^*-tree, we followed the sequence $(S, 0)$, $(B, 6)$, with $(A, 7)$ and not $(C, 7)$, which are equally weighted. We chose the node to the left side subtree. Had we chosen, $(C, 7)$ in place of $(A, 7)$, we would have reached to $(G, 9)$ as next node, which incidentally, was not a good choice.

15.3.4 Analysis of A^* Search

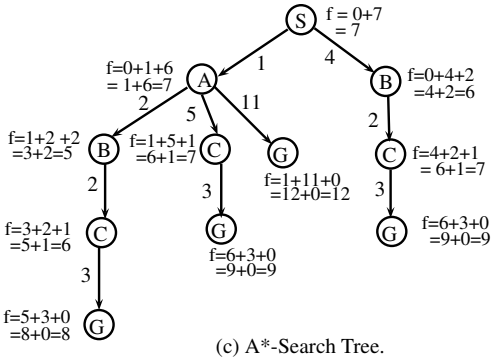
Let $h^*(n)$ be the *optimal cost* of the path from n to the goal node, and $g^*(n)$ is the optimal cost from start node to the node n, then$f^*(n)$ is *optimal* cost of path constrained to go through node n. The function $h^*(n)$ is undefined for node n that has no accessibility to goal node. Since, we intend to compute estimated minimum costs, let the corresponding figures for estimated costs are, $f(n)$, $g(n)$, and $h(n)$.



(a) Graph.

State	h
S	7
A	6
B	2
C	1
G	0

(b) Heuristic Table.



(c) A*-Search Tree.

Figure 15.4: Graph and A*-Search-tree.

Assume that cost from start node **S** to node n is $c(s, n)$. This is equivalent to a function g^* as

$$g^*(n) = c(s, n) \tag{15.3}$$

for all n accessible to **s**. Since $f^*(n)$ is optimal cost from start node **s** to goal node for any node n, we represent evaluation function $f^*(n)$ as

$$f^*(n) = g^*(n) + h^*(n) \tag{15.4}$$

Here $f^*(n)$ is the cost of an optimal path from **s** constrained to go through node n. In this case $f^*(s) = f(s)$ is the condition for actual cost of an unconstrained optimal path from **s** to goal node.

Assume that f is an estimate of f^*. Then, we can extend the optimal functions definition for actual cost as,

$$f(n) = g(n) + h(n) \tag{15.5}$$

where g and h are estimates of g^* and h^*, respectively. The $g(n)$ is cost of the path in the search tree from s to n computed by summing the arc costs encountered, while tracing the pointers back from n to s. This path is lowest cost path found so far by search algorithm. However, note that $g(n) \geq g^*(n)$.

The estimated value $h(n)$ for $h^*(n)$ can be obtained from the problem domain. For example, in the case of 8-puzzle problem or the 8-queen problem, this value is inverse of the number of tiles out of place with the goal or the inverse of number of queens giving checks to other queens. Even better value is, actual number of moves from n to goal.

Let us consider the specific cases of $f^*(n)$ as follows:

- When $h = 0$, then $g = d$ (the distance to goal in the search tree). This algorithm is called as **A**, and it is identical to breadth-first search (BFS).
- We claimed that the BFS algorithm is guaranteed to find the minimum path length to the goal node. If h is lower bound on h^*, i.e., $h(n) \leq h^*(n)$, for all n, then the algorithm will find an optimal path to a goal node. This algorithm is called as **A***.