CS 323: AI, QUIZ #2, Time 20 mts. DT. 30/01/2016. (Solution included.)

1. Find out the clauses for the following FOPL formula. [5]

   $\exists x \forall y \exists z (P(x) \Rightarrow (Q(y) \Rightarrow R(z)))$.

Ans. $\neg P(a) \vee \neg Q(y) \vee R(f(y))$

2. Determine whether the expression p and q unify. If so, give the *mgu*. The lowercase letters are variables, and upper are predicate, functions, and literals. [5]

$$p = f(x, f(u, x)),$$
$$q = (f(f(y, a), f(z, f(b, z)))).$$

Ans. $\{f(y, a)/x, z/u, b/y, a/z\}$

3. For the following, find the most general unifier if it exists: [5]

$$\{f(x, g(x)) = y, h(y) = h(v), v = f(g(z), w)\}.$$

Ans. $mgu = \{g(z)/x, g(x)/w\}$

4. Find out the valid and invalid substitutions: [5]

   (a) $\{f(z)/x, y/z\}$: Valid

   (b) $\{a/x, g(y)/y, f(g(b))/z\}$: Not Valid, as it causes recursion; $g(g(g(y)))$..

   (c) $\{y/x, g(b)/y\}$: Valid

   (d) $\{a/x, g(y)/x, f(g(b))/z\}$: not Valid as there are two different substitutions for same variable x

   (e) $\{g(y)/x, z/f(g(b))\}$: Not Valid as a variable has been substituted for literal (constant)

5. For the set $\{P(x), P(f(y))\}$, show that $\lambda = \{f(f(z))/x, f(z)/y\}$ is unifier, and $\theta = \{f(y)/x\}$ is mgu. [5]

Ans. We note that θ and λ are both unifying the expression $\{P(x), P(f(y))\}$. For θ to be an mgu, there should be some unifier μ, so that $\lambda = \theta\mu$. This is satisfied by $\mu = f(z)/y$. We note that,

$$\begin{aligned}\theta\mu &= \{f(y)/x\}\{f(z)/y\} \\ &= \{f(f(z))/x, f(z)/y\} \\ &= \lambda.\end{aligned}$$

This proves that θ is an mgu.