

Lecture 7: January 30, 2016

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7.1 Forward Chaining

In a forward chaining system, you start with the initial facts, and keep using the rules to draw new conclusions (or take certain actions), given those facts. Forward chaining systems are primarily *data-driven*. Whenever an if pattern is observed to match an assertion, the antecedent is *satisfied*. When all the *if* patterns of a rule are satisfied, the rule is *triggered*. When a triggered rule establishes a new assertion or performs an action, it is *fired*. This procedure is repeated until no more rules are applicable (figure ??).

The selection process is carried out in two steps:

1. *Pre-selection:* Determining the set of all the matching rules, also called the *conflict-set*.
2. *Selection:* Selection of a rule from the conflict set by means of a conflict resolving strategy.

7.2 Forward chaining Algorithm

A simple forward chaining algorithm 1 starts from the known facts in the knowledge-base, and triggers all the rules whose premises are the known facts, then adds the consequent of each into the knowledge-base. This process is repeated until the query is answered or until there is no conclusion generated to be added into the knowledge-base. We will use symbols θ, λ, γ to represent substitutions. The *unify* is a function unifies the newly generated assertion q' and the query α , and returns a unifying substitution λ if they are unified, else returns null.

Let α be the goal. The forward-chaining algorithm 1 picks up any sentence $s \in \Gamma$, where Γ is knowledge-base, and checks all possible substitutions θ for s . Let, the predicate form of s is $p_1 \wedge p_2 \wedge \dots \wedge p_n \rightarrow q$. On substituting θ , say, it results to $(p_1 \wedge p_2 \wedge \dots \wedge p_n)\theta = (p'_1 \wedge p'_2 \wedge \dots \wedge p'_n)$, such that p'_i 's $\in \Gamma$. Let $(p'_1 \wedge p'_2 \wedge \dots \wedge p'_n) \rightarrow q'$, such that $q' = q\theta$. This q' is added into *new* inference. If the inference q' and goal α unify, then their unifier λ is returned as the solution, else the algorithm continues.

Algorithm 1 Forward-chaining(Input: Γ, α) // α is a query, Γ is knowledge-base

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1: while True do
2:    $new = \{\}$ 
3:   for each sentence  $s \in \Gamma$  do
4:     Convert  $s$  into the format  $p_1 \wedge p_2 \wedge \dots \wedge p_n \rightarrow q$ 
5:     for each substitution  $\theta$  such that  $(p'_1 \wedge p'_2 \wedge \dots \wedge p'_n) \leftarrow (p_1 \wedge p_2 \wedge \dots \wedge p_n)\theta$  for some  $p'_i s \in \Gamma$  do
6:        $q' \leftarrow q\theta$ 
7:        $new \leftarrow new \cup \{q'\}$ 
8:        $\lambda \leftarrow unify(q', \alpha)$ 
9:       if  $\lambda$  is not null then
10:        return  $\lambda$ 
11:       end if
12:     end for
13:    $\Gamma \leftarrow \Gamma \cup new$  // add new inferences in knowledge-base
14: end for
15: end while
16: Return Fail

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The algorithm may fail to terminate in the case when the raised query has no answer. For example, every natural number can be generated by recursively applying successor operation on a natural number, and assuming that 0 is natural number. This will lead to indefinite loop for very large numbers.

$$\begin{aligned}
 & naturalnum(0). \\
 & \forall x[naturalnum(x) \rightarrow naturalnum(succ(x)).]
 \end{aligned}$$

The following example demonstrates the manual run of forward chaining algorithm.

Example 7.1 Produce the inference, given the knowledge-base $\Gamma = \{man(x) \rightarrow mortal(x), man(socrates)\}$ and query $\alpha = mortal(w)$, i.e., “Who is mortal?”

Solution. We follow the algorithm 1 manually and note that $p_1 \wedge \dots \wedge p_n = p_1 = man(x)$; $p'_1 = man(socrates)$, substitution $\theta = \{socrates/x\}$, and $q = mortal(x)$. Also,

$$\begin{aligned}
 q' &= q\theta \\
 &= mortal(x)\{socrates/x\} \\
 &= mortal(socrates).
 \end{aligned}$$

Also, $new = \{\} \cup \{q'\} = mortal(socrates)$.

On unification of α (i.e, $mortal(w)$), and q' , the unifier λ obtained is,

$$\begin{aligned}
 \lambda &= unify(q', \alpha) \\
 &= unify(mortal(socrates), mortal(w)) \\
 &= \{socrates/w\}.
 \end{aligned}$$

Hence, the answer for query is $w = \text{socrates}$. □

7.3 Backward Chaining Algorithm

The algorithm for backward chaining returns the set of substitutions (*unifier*) which makes the goal true. These are initially empty. The input to the algorithm is knowledge-base Γ , goals α , and current substitution θ (initially empty). The algorithm returns the substitution set λ for which the goal is inferred. The algorithm 2 is the backward-chaining algorithm.

Algorithm 2 Backward-chaining (Input: Γ, α, θ) // α is a query, Γ is knowledge-base, θ current substitution (initially empty), λ represent substitution set for the query to be satisfied (initially empty).

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1:  $\theta = \{\}, \lambda = \{\}$ 
2:  $q' \leftarrow \alpha\theta$ 
3: for each sentence  $s \in \Gamma$ , where  $s = p_1 \wedge p_2 \wedge \dots \wedge p_n \rightarrow q$  and  $\gamma \leftarrow \text{unify}(q, q') \neq \text{null}$  do
4:    $\alpha_{\text{new}} \leftarrow (p_1 \wedge p_2 \wedge \dots \wedge p_n)$ 
5:    $\theta \leftarrow \theta\gamma$ 
6:    $\lambda \leftarrow \text{backward-chaining}(\Gamma, \alpha_{\text{new}}, \theta) \cup \lambda$ 
7: end for
8: return  $\lambda$ 

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Example 7.2 Apply the backward-chaining algorithm for inferencing from a given knowledge-base.

Solution. Let $\Gamma = \{\text{man}(x) \rightarrow \text{mortal}(x), \text{man}(\text{socrates})\}$. Assume that it is required to infer answer for “Who is mortal?” That is, goal $\alpha = \text{mortal}(w)$, find w . The loop iterations in the backward-chaining algorithm 2 are as follows:

1st Iteration: Initially θ is empty, hence, $q' = \alpha\theta = \text{mortal}(w)$. From algorithm and knowledge-base Γ , the sentence is, $s = (\text{man}(x) \rightarrow \text{mortal}(x))$. Next, $\gamma = \text{unify}(\text{mortal}(x), \text{mortal}(w)) = \{w/x\}$. Also, the new goal, $\alpha_{\text{new}} = \text{man}(x)$. The new value of current substitution is, $\theta \leftarrow \theta\gamma = \{\}\{w/x\} = \{w/x\}$. Next, compute $\lambda = \text{backward-chaining}(\Gamma, \text{man}(x), \{w/x\}) \cup \lambda$, as a recursive call.

Recursive call: We apply the algorithm in a recursive mode, and get $q' = \text{man}(x)\{w/x\} = \text{man}(w)$. Next, the subgoal $\text{man}(w)$ (i.e., q') matches with other subgoal (it is a *fact*) $\text{man}(\text{socrates}) \in \Gamma$. Hence, $\gamma = \text{unify}(\text{man}(\text{socrates}), \text{man}(w)) = \{\text{socrates}/w\}$. Next, the new goal is, $\alpha_{\text{new}} = \text{man}(\text{socrates})$ and new substitution is:

$$\begin{aligned}
 \theta &= \theta\gamma \\
 &= \{w/x\}\{\text{socrates}/w\} \\
 &= \{\text{socrates}/x\}.
 \end{aligned}$$

In the next call of recursion at step 5, $q' = \alpha\theta = \text{man}(\text{socrates})\{\text{socrates}/x\}$, the substitution fails, as there is no x where “socrates” can be substituted. Hence, $q' = \text{null}$. Since γ is null, the *for loop* at step 3 terminates, and returned value of λ is $\{\text{socrates}/w\}$, i.e., $w = \text{socrates}$, or $\text{mortal}(\text{socrates})$. □