# Fourier Transform and its Applications

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# Outlines:

- Joseph Fourier
- Motivation
- Applications
- Introduction to Fourier transform
- Finding frequency cntents of a function
- Fitting in Complex Numbers and Exponentials
- Fourier series
- Use of ScLab – an open source software
- Resources for further reading

# Joseph Fourier

- While working on the flow of heat, French mathematician [ Jean Baptiste] Joseph Fourier (March 21, 1768 – May 16, 1830) discovered the equation for it that now bears his name.
- Fourier, the son of a tailor, born in Auxerre (France), was orphaned at the age of eight.
- Fourier was active politically during the upheaval of the French Revolution.
- In 1798 Fourier accompanied young General Napoleon Bonaparte on his Eastern expedition.
- After Fourier returned to Paris Napoleon made him an offer that was actually a command to serve as the Prefect

# Motivation

- Ninety percent of all physics is concerned with vibrations and waves of one sort or another.
- An oscilloscope will display a graph of pressure against time, $F(t)$, which is periodic.
- This waveform is not a pure sinusoid
- The waveform can be analysed to find the amplitudes of the overtones,

# Motivation...

$A(v)$ is the Fourier transform of $F(t)$.

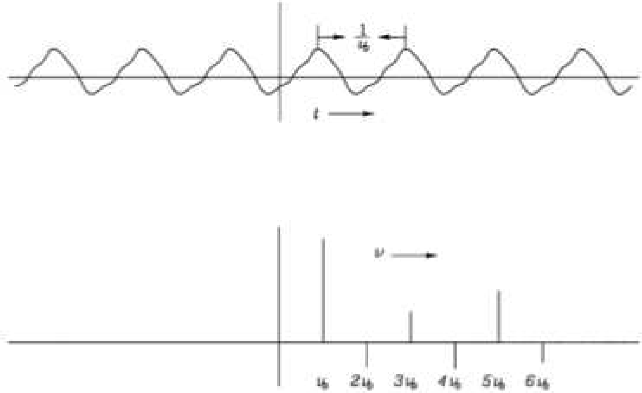


Fig. 1.1. The spectrum of a steady note: fundamental and overtones.

# Applications

Actually it is the modular transform, but at this stage that is a detail. Suppose that the sound is not periodic

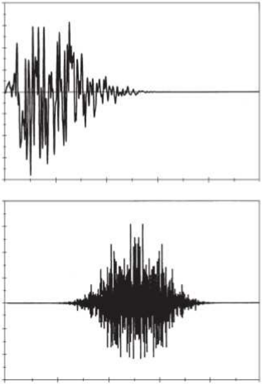


Fig. 1.2. The spectrum of a crash: all frequencies are present.

Some uses of Fourier tran.:

- selection of a valuable violin;
- analysis of the sound of an aero-engine to detect a faulty gear-wheel;
- an electrocardiogram to detect a heart defect;
- light curve of a periodic variable star to determine the underlying physical causes of the variation,

# Applications...

Methods based on the Fourier transform are used in virtually all areas of engineering and science and by virtually all engineers and scientists. For starters:

- Circuit designers
- Spectroscopists
- Crystallographers
- Anyone working in signal processing and communications
- Anyone working in imaging

- Fourier analysis was originally concerned with representing and analyzing periodic phenomena, via Fourier series, and later with extending those insights to nonperiodic phenomena, via the Fourier transform.
- You can analyze the signal either in the time (or spatial) domain or in the frequency domain.

# Introduction Fourier Transform

- The Fourier transform is among the most widely used tools for transforming data sequences and functions (single or multi-dimensional), from what is referred to as the time domain to the frequency domain.
- Functions as Combinations of Sinusoids: Any continuous, periodic function can be represented as a linear combination of sines and cosines. A sine is a function of the form:

$$A\ sin(2\pi\omega t+\phi),$$

where $A$ - *amplitude*, $\omega$ - *frequency*, and $\phi$ - *phase*, which is used for getting values other than 0 at $t=0$. A cosine function has exactly the same components as the sine, and can be viewed as a shifted sine (i.e., a sine with phase $\pi/2$).

# Introduction Fourier Transform...

- Thus, given a function $f(t)$, we can usually rewrite it (or at least approximate it), for some $n$ as:

$$f(t) = \sum_{k=1}^{n} (A_k cos(2\pi\omega_k t) + B_k sin(2\pi\omega_k t)) \tag{1}$$

- Both sines and cosines are combined, rather than only sines, to allow the expression of functions for which $f(0) \neq 0$, in a way that is simpler than adding the phase to the sine in order to make it into a cosine.
- As an example of a linear combination of sinusoids consider the function:

$$f_1(t) = 0.5 sin\pi t + 2 sin 4\pi t + 4 cos 2\pi t \tag{2}$$

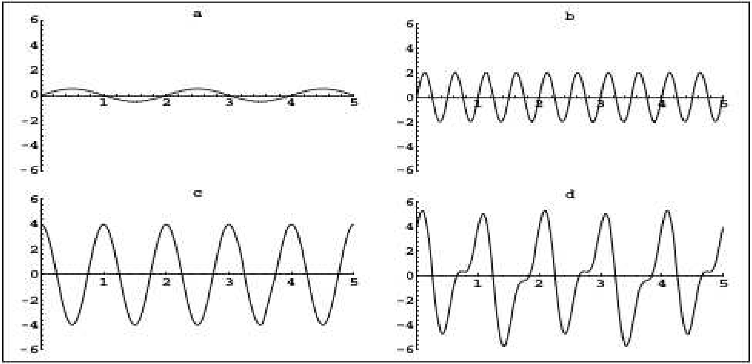



**Figure 1:** A plot of $f_1(t)$, $(d)$, and its components $(a, b, c)$, for $t = 0..5$

The function $f_1(t)$ consists of sines and cosines of 3 frequencies.

# Frequency contents of the function $f_1(t)$

| k | Frequency ($\omega_k$) | Cosine amplitude ($A_k$) | Sine Amplitude($B_k$) |
|---|---|---|---|
| 1 | 1/2 | 0 | 1/2 |
| 2 | 2 | 0 | 2 |
| 3 | 1 | 4 | 0 |

- The representation of a periodic function (or of a function that is defined only on a finite interval) as the linear combination of sines and cosines, is known as the Fourier series expansion of the function.
- The Fourier transform is a **tool** for obtaining such *frequency* and *amplitude* information for sequences and functions, which are not necessarily periodic.
- These sequences are just a special case of functions.

# Fitting in Complex Numbers and Exponentials

$$e^{i\theta} = cos(\theta) + i\ sin(\theta), \quad e^{-i\theta} = cos(\theta) - i\ sin(\theta)$$

Hence, $cos(\theta) = \frac{e^{i\theta}+e^{-i\theta}}{2}$, $sin(\theta) = \frac{e^{i\theta}-e^{-i\theta}}{2i}$
Thus,

$$f(t) = \sum_{k=1}^{n} [\frac{A_k}{2}(e^{2\pi i\omega_k t} + e^{-2\pi i\omega_k t}) + \frac{B_k}{2i}(e^{2\pi i\omega_k t} - e^{-2\pi i\omega_k t})]$$

where $C_k = \frac{A_k - iB_k}{2}$ for $k > 0$, and
$C_k = \frac{A_k + iB_k}{2}$ for $k < 0$, and
$C_0 = 0$, and
$\omega_k = -\omega_{-k}$, for $k < 0$, and

$$f(t) = \sum_{k=-n}^{n} [C_k e^{2\pi i\omega_k t}]$$

For a steady note, description requires only the fundamental frequency, its amplitude and the amplitudes of its harmonics.

$$F(t) = a_0 + a_1 cos(2\pi v_0 t) + b_1 sin(2\pi v_0 t) + a_2 cos(4\pi v_0 t) + b_2 sin(4\pi v_0 t) + b_3 cos(6\pi v_0 t) + .... \quad (3)$$

where $v_0$ is the fundamental frequency of the note.

$$F(t) = \sum_{n=-\infty}^{\infty} a_n cos(2\pi n v_0 t) + b_n sin(2\pi n v_0 t) \quad (4)$$

The $-\infty$ is for mathematical symmetry. Since $cosx = cos(-x)$, $sinx = -sin(-x)$,

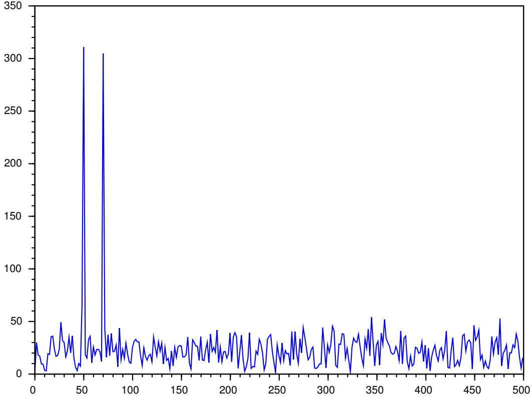
$$F(t) = A_0/2 + \sum_{n=1}^{\infty} A_n cos(2\pi n v_0 t) + B_n sin(2\pi n v_0 t) \quad (5)$$

where $A_n = a_n + a_n$, $B_n = b_n - b_{-n}$

# Fast Fourier Transform using SciLab

$$s = sin(2*\pi*50*t) + sin(2*\pi*70*t + \pi/4) + grand(1, N, 'nor', 0, 1)$$

$$y = fft(s)$$



# Resources

- http://mathfaculty.fullerton.edu/mathews/c2003/FourierTransformBib/Links/ FourierTransformBib_lnk_1.html
- http://mathworld.wolfram.com/FourierSeries.html
- https://ocw.mit.edu/courses/mathematics/18-03sc-differential-equations-fall-2011/unit-iii-fourier-series-and-laplace-transform/fourier-series-basics/
- https://math.stackexchange.com/questions/195071/need-a-crash-course-in-fourier-analysis-recommend-resources