Parallel Processing technology (Interconnection networks)

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Prof. K.R. Chowdhary *: Professor of CS*

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4.1 Basic Properties of Interconnection Networks

We can classify interconnection networks in many ways and characterize them by certain parameters. For defining these parameters, *graph theory* is the most elegant mathematical framework. More specifically, an interconnection network can be modeled as a graph $G = (V, E)$, where V (for vertices) is a set of communication nodes and E (set of edges) is a set of communication links (or, channels) between the communication nodes. Based on this graph-theoretical view of interconnection networks, we can define parameters that represent both *topological* properties and *performance* properties of interconnection networks.

Following are The topological properties of interconnection networks, that are defined by graph-theoretical notions:

- *Node Degree.* The node degree is the number d of channels through which a communication node is connected to other communication nodes. Note that node degree includes only the ports for the network communication, although a communication node also needs ports for the connection to the processing element(s) and ports for service or maintenance channels.
- *Regularity.* An interconnection network is said to be regular if all communication nodes have the same node degree; that is, there is a $d > 0$ such that every communication node has node degree d.
- *Symmetry.* An interconnection network is said to be symmetric if all communication nodes possess the "same view" of the network; that is, there is a *homomorphism* that maps any communication node to any other communication node. In a symmetric interconnection network, the load can be evenly distributed through all communication nodes, thus reducing congestion problems. Many real implementations of interconnection networks are based on symmetric regular graphs because of their fruitful topological properties that lead to a simple routing and fair load balancing under the uniform traffic.
- *Diameter.* In order to move from a source node to a destination node, a packet must traverse through a series of elements, such as routers or switches, that together comprise a path (or, route) between the source and the destination node. The number of communication nodes traversed by the packet along this path is called the *hop count.* In the best case, two nodes communicate through the path which has the minimum hop count, l, taken over all paths between the two nodes. Since l may vary with the

source and destination nodes, we also use the average distance, l_{avg}, which is average l taken over all possible pairs of nodes. An important characteristic of any topology is the diameter, l_{max}, which is the maximum of all the minimum hop counts, taken over all pairs of source and destination nodes.

- *Path diversity.* In an interconnection network, there may exist multiple paths between two nodes. In such case, the nodes can be connected in many ways. A packet starting at source node will have at its disposal multiple routes to reach the destination node. The packet can take different routes (or even different continuations of a traversed part of a route) depending on the current situation in the network. An interconnection network that has high path diversity offers more alternatives when packets need to seek their destinations and/or avoid obstacles.

- *Scalability.* Scalability is (i) the capability of a system to handle a growing amount of work, or (ii) the potential of the system to be enlarged to accommodate that growth. The scalability is important at every level. For example, the basic building block must be easily connected to other blocks in a uniform way. Moreover, the same building block must be used to build interconnection networks of different sizes, with only a small performance degradation for the maximum-size parallel computer. Interconnection networks have important impact on scalability of parallel computers that are based on the LMM or MMM multiprocessor model. To appreciate that, note that scalability is limited if node degree is fixed.

4.2 Classification of Interconnection Networks

Interconnection networks can be classified into direct and indirect networks. Here are the main properties of each kind.

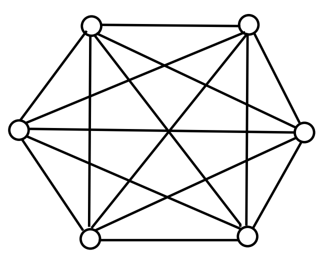

Figure 4.1: A fully connected network with 6 nodes

Direct Networks A network is said to be direct when each node is directly connected to its neighbors. How many neighbors can a node have? In a fully connected network, each of the $n = |N|$ nodes is directly connected to all the other nodes, so each node has $n - 1$ neighbors. (See Fig. 4.1). Since such a network has $\frac{1}{2}n(n-1) = \theta(n^2)$, direct connections, it can only be used for building systems with small numbers n of nodes. Here θ is called complexity of the network.

When n becomes large, each node is directly connected to a proper subset of other nodes, instead of all nodes, and the communication to the remaining nodes is achieved by routing messages through these intermediate nodes. An example of such a direct interconnection network is the hypercube.

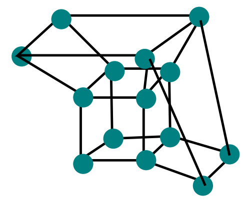

Figure 4.2: A hypercube. Each node represents a processor unit with local memory

Indirect Networks An indirect network connects the nodes through switches. Usually, it connects processing units on one end of the network and memory modules on the other end of the network. The simplest circuit for connecting processing units to memory modules is the *fully connected* crossbar switch (Fig. 4.3). Its advantage is that it can establish a connection between processing units and memory modules in an arbitrary way.

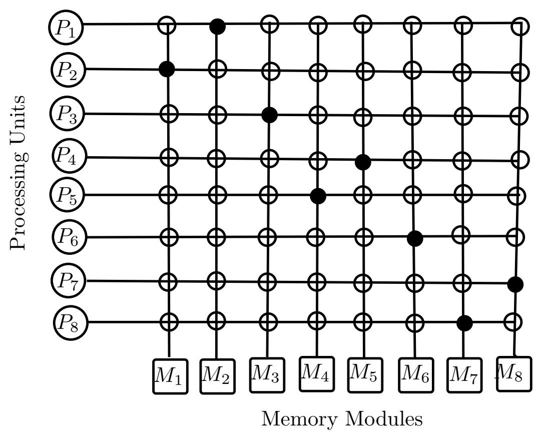

Figure 4.3: A fully connected crossbar switch connecting 8-nodes

At each intersection of a horizontal and vertical line is a cross-point. A cross-point is a small switch that can be electrically opened ($\circ$) or closed ($\bullet$), depending on whether the horizontal and vertical lines are to be connected or not. In Fig. 4.3 we see eight cross points closed simultaneously, allowing connections between the pairs $(P_1, M_2), (P_2, M_1)$, $(P_3, M_3), (P_4, M_5), (P_5, M_4)$, $(P_6, M6)$, (P_7, M_8) and (P_8, M_7) at the same time. Many other combinations are also possible.

Unfortunately, the fully connected crossbar has too large complexity to be used for connecting large numbers of input and output ports. Specifically, the number of cross points grows as $p.m$, where p and m are the numbers of processing units and memory modules, respectively. For $p = m = 1000$ this amounts to a million cross-points which is not feasible. However, for medium-sized systems, a crossbar design is workable, and small fully connected crossbar switches are used as basic building blocks within larger switches and routers.

This is why indirect networks connect the nodes through many switches. The switches themselves are usually connected to each other in stages, using a regular connection pattern between the stages. Such indirect networks are called the multi-stage interconnection networks.

Indirect networks can be further classified as follows:

- A *non-blocking network* can connect any idle source to any idle destination, regardless of the connections already established across the network. This is due to the network topology which ensures the existence of multiple paths between the source and destination.
- A *blocking rearrangeable* networks can rearrange the connections that have already been established across the network in such a way that a new connection can be established. Such a network can establish all possible connections between inputs and outputs.
- In a *blocking network*, a connection that has been established across the network may block the establishment of a new connection between a source and destination, even if the source and destination are both free. Such a network cannot always provide a connection between a source and an arbitrary free destination.

The distinction between direct and indirect networks is less clear nowadays. Every direct network can be represented as an indirect network since every node in the direct network can be represented as a router with its own processing element connected to other routers. However, for both direct and indirect interconnection networks, the full crossbar, which is used as an ideal switch, is the heart of the communications.

Review Questions

1. Define a *Regular* interconnection network.
2. A fully connected network is a direct network or indirect network?
3. A fully connected crossbar switch is a direct or indirect network?
4. What are the advantages of fully connected crossbar switch network?
5. What are the disadvantages of a fully connected crossbar switch network?
6. Why the distinction between direct and indirect network is less clear now a day?

Exercises

1. How the local memory is beneficial in bus system to schedule bus contention?
2. Find out the diameters of the networks shown in Figs.: 4.1, 4.3, and 4.2.
3. In a $n \times n$ fully connected crossbar switch network where there are n processors and n memory modules connected in one-to-one fashion, how many total connections switches are to be provided? What is its communication complexity?
4. In the above question, how many ways these switches can be connected?