Universal Turing Machine

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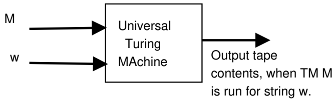
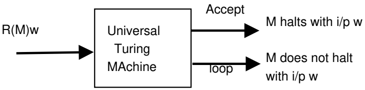
TM Simulations & Universal TM

- A 3-tape TM, 2D-TM, and NDTM can be simulated by a standatd TM. Also, A TM can be also simulated by a TM.
- Let Input $= [M, w]$ to a TM M'. Output of M' is what, when M runs with input w. M' is Universal Turing machine (UTM).
- A UTM can be designed to simulate the computations of an arbitrary TM M. To do so, input to UTM must contain representation of both - machine M and input w to be processed by M.



TM Simulation on another TM

- Let there is TM M that accepts by halting. The UTM M' for this is:
  with Input string $= R(M)w$, where $R(M)$ is representation of M.
- Output-1: Accept (indicates that M halts with input w), output-2: loops, i.e., M does not halt with input w, i.e. computation of M does not terminate.
- The machine M' is called universal TM, as computation of any Turing machine can be simulated by M'.



Design a string representation of a TM M

Because of the ability to encode arbitrary symbols as strings over $\{0, 1\}$, we consider Turing machine with inputs $\{0, 1\}$ and tape symbols $\Gamma = \{0, 1, B\}$

Encoding of elements of M:

Symbol	Encoding
0	1
1	11
B	111
q_0	1
q_1	11
...	...
q_n	1^{n+1}
L	1
R	11

- The states of M are assumed to be $\{q_0, q_1, \dots, q_n\}$. TM M is defined by its transition function:

$$\delta(q_i, a) = (q_j, b, d)$$

  where,
  $q_i, q_j \in Q; a, b \in \Gamma; d \in \{L, R\}$
- Let $en(z)$ denote the encoding of z. Thus, transition $\delta(q_i, a) = (q_j, b, d)$ is encoded by string: $en(q_i)0en(a)0en(q_j)0en(b)0en(d)$. The symbol 0 separates the different components of δ.

Encoding of elements of M

Representation of machine M is constructed from encoded transitions. Two consecutive 0s separate transitions. Beginning and end of complete representation are defined by three 0s.

Consider the Transitions:

Transition	Encoding
$\delta(q_0, B) = (q_1, B, R)$	101110110111011
$\delta(q_1, 0) = (q_0, 0, L)$	1101010101
$\delta(q_1, 1) = (q_2, 1, R)$	110110111011011
$\delta(q_2, 1) = (q_0, 1, L)$	1110110101101

- The machine M is represented by string: 000101110110111011 00110101010100110110111011011001110110101101000

  Simulation of M on Universal TM M'

  Verification of representation of M: TM can be constructed to check whether an arbitrary string $u \in \{0,1\}^*$ is encoding of deterministic TM M. Computations examines whether 000 is prefix, followed by finite sequences of encoded transitions are separated by 00s, then finally 000.
- M is deterministic if $Q \times \Gamma$ in every encoded transition is unique.

Simulation of TM M on 3-tape DTM M'

- Tape-1 holds $R(M)w$. Tape-3 simulates computations of of M for input w. Tape-2 acts as working tape.
- If input u is not of the form $R(M)w$ for deterministic TM M and string w on tape-1, the M' moves to right forever.

1. w is copied from tape-1 to 3, with tape head at begin of w. $\therefore$ tape-3 is initial configuration of M with input w.
2. Encoding of q_0, i.e., 1 is written tape-2. (for future steps, we call it q_j).
3. Transition of M is simulated on tape-3. The next transition is determined by symbol scanned on tape-3 and state encoded on tape-2. Let these are a and q_i.
4. Tape-1 is scanned for a and q_i as first two components of a transition. If not found, M' halts by rejecting input.
5. If tape-1 consists the encoded information for above, i.e., $\delta(q_i, a) = (a_j, b, d)$, then
   (a) q_i replaced by q_j on tape-2.
   (b) b is written on tape 3, and tape head on tape-3 is moved for direction given in d.
6. Go back to step 2, and carry on computation by simulating M.