4CS4-6: Theory of Computation (TM as Language Recognizer)

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19.1 TM as Language Recognizer

Example 19.1 *Construct a TM that accepts the language* $L = \{a^n b^n \mid n \geq 0\}$.

Let $M = (Q, \Sigma, \Gamma, \delta, s, H)$ be a Turing machine, which accepts the language L. Let us assume that the string $w = a^n b^n$ exists on the tape, followed by infinite number of blank characters B. The working of M to recognize w is explained in following steps:

1. M replace left most a by X, and then head moves to right until it encounters left most b.
2. Replaces this b by Y, and then moves left to find the right most X. Then moves one step right to left most a.
3. Repeat Step 1 and 2 in order, i.e., 1, 2, 1, 2, ...
4. When searching for b, if M finds a blank character B (i.e., $|a^n| = |b^n|$) then M halts without accepting.
5. If a is not found but it finds b ($a^n \neq b^n$), then M halts without accepting.
6. After changing b to Y, if M finds no more a then it checks that no more b remains. If this is true then $a^n b^n$ is accepted by M (i.e., $|a^n| = |b^n|$).

Based on the above procedure, M can be specified as given below.

$$
\begin{aligned}
Q &= \{q_0, q_1, q_2, q_3, q_4\} \\
\Sigma &= \{a, b\} \\
\Gamma &= \{\triangleright, a, b, X, Y, B\} \\
s &= q_0 \\
H &= \{q4\}
\end{aligned}
$$

The transition function for this machine is given below:

Fig. 19.1 shows a transition diagram for this TM accepting language $L = \{a^n b^n \mid n \geq 0\}$.

Table 19.1: Transition table for $a^n b^n$ Language

state	a	b	X	Y	B
q_0	(q_1, X, R)			(q_3, Y, R)	(q_4, B, L)
q_1	(q_1, a, R)	(q_2, Y, L)		(q_1, Y, R)	
q_2	(q_2, a, L)		(q_0, X, R)	(q_2, Y, L)	
q_3				(q_3, Y, R)	(q_4, B, L)

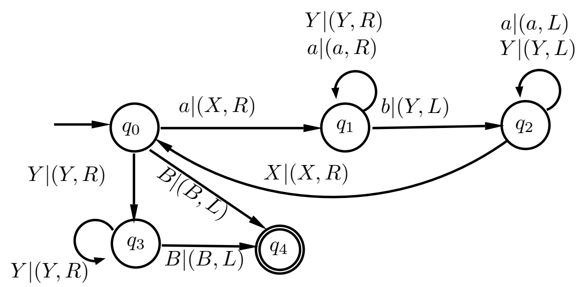

Figure 19.1: Transition diagram for TM accepting Language $L = \{a^n b^n \mid n \geq 0\}$

Let $w = aabb$. The following are sequence of moves, which leads to the acceptance of w. A character B after $aabb$ indicates end of the input string.

$$\begin{aligned} q_0aabbB &\vdash Xq_1abbB \vdash Xaq_1bbB \vdash Xaq_2YbB \vdash Xq_2aYbB \\ &\vdash q_2XaYbB \vdash Xq_0aYbB \vdash XXq_1YbB \vdash XXYq_1bB \\ &\vdash XXq_2YYB \vdash Xq_2XYYB \vdash XXq_0YYB \vdash XXYq_3YB \\ &\vdash XXYYq_3B \vdash XXYq_4YB \end{aligned}$$

The reader may verify that strings aab, $abab$, abb etc., are not accepted.

Example 19.2 *Finding complexity of TM for $L = \{a^n b^n \mid n \geq 0\}$.*

The time (number of steps) taken by the 1-tape Turing machine for this can be calculated as follows:

- Assume that R/W head was initially pointing to input symbol a of a^n. In the first run, the head moves total $n-1$ steps in a's and 1 step in b (total n steps) in forward direction, and equal in reverse, with total $2n$ steps.
- This is repeated for n-times, making total of $n * n = n^2$ steps.
- Next, there are total n-steps for transitions from q_0 to q_3, plus for loop at q_3 state. And finally one step for q_3 to q_4 transition.
- Thus total transitions are $n^2 + n + 1$ for string $a^n b^n$. For an input $|w| = n$, this value is $(n^2 + n + 1)/2$. This gives a time of $(n^2 + n + 1)/2$, or time complexity of $O(n^2)$.

Example 19.3 *Show that the language* $L = \{a^n b^n c^n \mid n \geq 0\}$ *is accepted by a TM.*

We have just seen that language $a^n b^n$ can be accepted by a TM. In that process $a^n b^n$ gets converted to $X^n Y^n$ before the TM halts.

To design a TM to accept $a^n b^n c^n$, we use the almost the previous technique and perform the following conversions:

1. $aaabbbccc$ is converted to $XaaYbbZcc$,
2. After the next operation, the tape contents are $XXaYYbZZc$,
3. Finally, the tape contents are $XXXYYYZZZ$.

The above can be carried out by adding one more state after q_2, say q_2'. Hence, this proves that language $a^n b^n c^n$ is accepted by a TM using the approach similar to method of example 19.1.