32002: AI (Default Reasoning) **Spring 2014**

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21.1 Nonmonotonic reasoning

Nonmonotonic logic is the study of those ways of inferring additional information from given information that do not satisfy the monotonicity property satisfied by all methods based on classical (mathematical) logic. In Mathematics, if a conclusion is warranted on the basis of certain premises, no additional premises will ever invalidate the conclusion. However, in everyday life, it seems clear that we, human beings, draw sensible conclusions from what we know and that, on the face of new information, we often have to take back previous conclusions, even when the new information we gathered in no way made us want to take back our previous assumptions. It is most probable that intelligent automated systems will have to do the same kind of (nonmonotonic) inferences.

21.2 Default Reasoning

One of the features of commonsense reasoning that makes it different from traditional mathematical proofs is the use of defaults. A default is a proposition that is postulated to be true in the absence of information to the contrary. For instance, an intelligent agent may assume by default that his observation correctly reflects the state of the world, and be prepared to retract this assumption in the face of evidence that be is in error.

When an intelligent system (either computer-based or human) tries to solve a problem, it may be able to rely on complete information about this problem, and its main task is to draw the correct conclusions using classical reasoning. In such cases, classical predicate logic may be sufficient. However, in many situations the system has only incomplete information at hand, be it because some pieces of information are unavailable, or because it has to respond quickly and does not have the time to collect all relevant data.

Classical logic indeed has the capacity to represent and reason with certain aspects of incomplete information. But there are occasions where additional information needs to be "filled in" to overcome the incompleteness, because certain decisions must be made. In such cases the system has to make some plausible conjectures, which in the case of default reasoning are based on *rules of thumb*, called *defaults*. For example, an emergency doctor has to make some conjectures about the most probable causes of the symptoms observed. Obviously, it would be inappropriate to await the results of possibly extensive and time-consuming tests before beginning with treatment.

When decisions are based on assumptions, these may turn out to be wrong in the face of additional information that becomes available; i.e., medical tests may lead to a modified diagnosis. The phenomenon of having to take back some previous conclusions is called non-monotonicity; it says that if a statement φ follows from a set of premises M and $M \subseteq M'$, then φ does not necessarily follow from M' (see example 21.1). Default Logic provides formal methods to support this kind of reasoning.

Example 21.1 *Let*

$$\begin{aligned}
&M = \{(\forall x)(bird(x) \Rightarrow fly(x)),\\
&(\forall y)(penguin(y) \Rightarrow bird(y)),\\
&(\forall z)(penguin(z) \Rightarrow \neg fly(z)),\\
&bird(tweety)\}.
\end{aligned}$$

Given these, we have $M \models fly(tweety)$, and $M' = M \cup \{penguin(tweety)\}$ is inconsistent. We would expect the inference

$$M \cup \{penguin(tweety)\} \models \neg fly(tweety), \tag{21.1}$$

making $fly(tweety)$ a defeasible consequent. □

21.2.1 Notion of a default

A rule used by football organizers in Kashmir valley might be: "A football game shall take place, unless there is snow in the stadium." This rule of thumb is represented by the default

$$\frac{football : \neg snow}{takesPlace} \tag{21.2}$$

The interpretation of the default is as follows: If there is no information that there will be snow in the stadium, it is reasonable to assume $\neg snow$ and conclude that the game will take place (so preparations can proceed). But if there is a heavy snowfall during the night before the game is scheduled, then this assumption can no longer be made. Now we have definite information that there is snow, so we cannot assume $\neg snow$, therefore the default cannot be applied. In this case we need to refrain from the previous conclusion (i.e., the game will take place), so the reasoning is nonmonotonic.

Before proceeding with more examples, let us first explain why classical logic is not appropriate to model this situation. Of course, we could use the rule:

$$football \land \neg snow \rightarrow takesPlace. \tag{21.3}$$

The problem with this rule is that we have to definitively establish that there will be no snow in the stadium before applying the rule. But that would mean that no game could be scheduled in the winter. It is important to understand the difference between having to know that it will not snow, and being able to assume that it will not snow. Defaults support the drawing of conclusions based upon assumptions.

Perhaps the best example is the following main principle of justice in the courts of Law: "In the absence of evidence to the contrary assume that the accused is innocent." In default form:

$$\frac{accused(X) : innocent(X)}{innocent(X)} \tag{21.4}$$

The interpretation of rule 21.4 is that if $accused(X)$ is known, and there is no evidence that $innocent(X)$ is false, then $innocent(X)$ can be inferred.

A *default theory* T is a pair (W, D) consisting of a set W of first-order predicate logic formulas (called the facts or axioms or belief set of T) and a countable set D of default rules. A default rule δ has the form

$$\delta = \frac{\varphi : \psi_1, \ldots, \psi_n}{\chi} \tag{21.5}$$

where $\varphi, \psi_1, \ldots, \psi_n, \chi$, are closed predicate logic formulae, and $n > 0$, The formula φ is called the *prerequisite*, $\psi_1, \ldots, \psi_n$ are the *justifications*, and χ the *consequent* of δ .

One point that needs some discussion is the requirement that the formulae in a default be ground. This implies that formula like 21.4 is not a default according to the definition above. Let us call such rules of inference open defaults. An open default is interpreted as a default schema, meaning that it represents a set of defaults (this set may be infinite).

A default schema looks like a default, the only difference being that $\varphi, \psi_1, \ldots, \psi_n, \chi$ are arbitrary predicate logic formulas (i.e., they may contain free variables). A default schema defines a set of defaults, namely

$$\frac{\varphi\sigma : \psi_1\sigma, \ldots, \psi_n\sigma}{\chi\sigma} \tag{21.6}$$

for all *ground substitutions* σ that assign values to all free variables occurring in the schema. That means free variables are interpreted as being universally quantified over the whole default schema. Given a default schema

$$\frac{bird(X) : flies(X)}{flies(X)} \tag{21.7}$$

and the facts $bird(tweety)$ and $bird(sam)$, the *default theory* is represented as

$$\begin{aligned} T &=< W, D > \\ &=< \{bird(tweety), bird(sam)\}, \\ &\quad \{[bird(tweety) : flies(tweety)]/flies(tweety), \\ &\quad [bird(sam) : flies(sam)]/flies(sam)\} > . \end{aligned}$$

21.3 Algorithm for Default Reasoning

The algorithm 1 for default reasoning provides extensions to the formula set W. Let $T =< W, D >$ be a closed default theory with a finite set of default rules D and formula W. Let P be the set of all permutations of elements of default set D. If $P = \{\}$, i.e., $D = \{\}$, or if W is inconsistent, then return default theory $Th(W)$ as the only extension of T. The set of justifications and beliefs are indicated by *JUST* and *BELIEF*, respectively.

Algorithm 1 Algorithm for Default Reasoning.

$P =$ All permutations of elements of D
while $P \neq \{\}$ **do**
  Take a permutation, $perm = \{d_1, \ldots, d_n\} \in P$
  $P = P - \{perm\}$
  [*initialization*]
  $BELIEF = W, JUST = \{\}$
  [Application of defaults and consistency test]
  for $i = 1$ to n **do**
    [assume $d_i = \frac{A_i : B_i}{C_i}$]
    if $(BELIEF \vdash A_i) \wedge (BELIEF \nvdash \neg B_i)$ **then**
      $BELIEF = BELIEF \cup \{C_i\}$
      $JUST = JUST \cup \{B_i\}$
      if $(\exists A), A \in JUST$ such that $BELIEF \vdash \neg A$ **then**
        exit the algorithm
      end if
    end if
  end for
end while
Return Th(BELIEF)
End

Example 21.2 *Find extensions of following default theory:*

$$T =< W, D >=< \{R(n) \wedge Q(n)\}, \{\frac{R(x) : \neg P(x)}{\neg P(x)}, \frac{Q(x) : P(x)}{P(x)}\} >$$

Solution: First consider the permutation of D as

$$(d_1, d_2) = \{\frac{R(x) : \neg P(x)}{\neg P(x)}, \frac{Q(x) : P(x)}{P(x)}\} > .$$

At begin, we initialize $BELIEF = \{R(n) \wedge Q(n)\}, JUST = \{\}$. As per algorithm 1, $d_1 = \frac{A_1 : B_1}{C_1} = \frac{R(x):\neg P(x)}{\neg P(x)}$. From algorithm, we note that $BELIEF \vdash R(n)$, and $BELIEF \nvdash \neg(\neg P(n))$. Thus, we add C_i, i.e., $\neg P(n)$ in $BELIEF$. Also, add $\neg P(n)$ into $JUST$. We also note that, the justification does not negate the $BELIEF$, hence it is consistent.

Next we repeat the loop of algorithm 1, for $i = 2$: $(d_2) = \frac{A_2 : B_2}{C_2} = \frac{Q(x):P(x)}{P(x)}$. Now, the $BELIEF = \{R(n) \wedge Q(n), \neg P(n)\}$. We note that $BELIEF \vdash A_2$ (i.e., $Q(n)$), and $BELIEF \nvdash \neg B_2$ does not hold. This is because $\neg B_2 = \neg P(n)$, and already $\neg P(n) \in BELIEF$. Hence the belief is stable. This is shown in table below.

Iteration	BELIEFS	JUSTIFICATIONS	Consistency Test
Initial	$R(n) \wedge Q(n)$	$\{\}$	
$i = 1, d_1$	$\neg P(n)$	$\neg P(n)$	OK
$i = 2, d_2$	Stable		

When same thing is repeated for other permutation:

$$(d_2, d_1) = \{\frac{Q(x) : P(x)}{P(x)}, \frac{R(x) : \neg P(x)}{\neg P(x)}\},$$

we get the table as follows:

Iteration	BELIEFS	JUSTIFICATIONS	Consistency Test
Initial	$R(n) \wedge Q(n)$	$\{\}$	
$i = 1, d_2$	$P(n)$	$P(n)$	OK
$i = 2, d_1$	Stable		

In conclusion, the T has two extensions: $Th(\{R(n) \wedge Q(n), \neg P(n)\})$, and $Th(\{R(n) \wedge Q(n), P(n)\})$. □

References

[1] Chowdhary K.R. (2020) Logic and Reasoning Patterns. In: Fundamentals of Artificial Intelligence. Springer, New Delhi. https://doi.org/10.1007/978-81-322-3972-7_6