32002: AI (Propositional Logic & Inference Rules)	**Spring 2014**

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5.1 Syntax and Semantics

Syntax is name given to a correct structure of a statement. It is meaning associated with the expression. Its mapping to the real-world situation is *semantics*. The semantics of a language defines the truth of each sentence with respect to each possible *world*. For example, the usual semantics for interpretation of the statement $(p \vee q) \wedge r$ is true in a *world* where either p or q or both are *true* and r is *true*. Different worlds can be all the possible sets of *truth* values of p, q, r. The truth values are simply the assignment to these variables, and not necessarily the values which are only *true*. For example, $(0, 0, 1)$ and $(1, 0, 1)$ are the worlds for the expression $(p \vee q) \wedge r$.

5.1.1 Models

Logicians typically think in terms of *Models*, which are formally structured worlds with respect to which truth can be evaluated. We say m is model of a sentence x if x is true in m. Let $M(x)$ is set of all models of x, and S is knowledge-base. Then $S \models x$ if and only if $M(S) \subseteq M(x)$.

5.1.2 Interpretation of Expressions

If ϕ and ψ are formulas of set $\mathcal{P}$, and $\mathcal{I}$ is an interpretation of $\mathcal{P}$ then:

- A sentence ϕ of propositional logic is true under an interpretation $\mathcal{I}$ iff $\mathcal{I}$ assigns the truth value T to that sentence. If a sentence is true under an interpretation, then that interpretation is called a *model* of that sentence.
- ϕ is false under an interpretation $\mathcal{I}$ iff ϕ is not true under $\mathcal{I}$.
- A sentence is logically valid iff it is true under every interpretation. $\models \phi$ means that ϕ is logically valid. All the formulas in table 5.1 are valid.
- A sentence ψ of propositional logic is a logical consequence of a sentence ϕ iff there is no interpretation under which ϕ is true and ψ is false. This is represented by $\phi \models \psi$. In the formula $((p \rightarrow q) \wedge p) \vdash q$, the q is logical consequence of $((p \rightarrow q) \wedge p)$.
- A sentence of propositional logic is *consistent* iff it is true under at least one interpretation. It is *inconsistent* if it is not consistent.

Table 5.1: Inference Rules.

Rule	**Formula**	**Description**
Modus Ponens	$((p \rightarrow q) \wedge p) \vdash q$	If p then q; p; therefore q
Modus Tollens	$((p \rightarrow q) \wedge \neg q) \vdash \neg p$	If p then q; not q; therefore not p
Hypothetical Syllogism	$((p \rightarrow q) \wedge (q \rightarrow r)) \vdash (p \rightarrow r)$	if p then q; if q then r; therefore, if p then r
Disjunctive Syllogism	$((p \vee q) \wedge \neg p) \vdash q$	Either p or q, or both; not p; therefore, q

5.2 Reasoning Patterns

The reasoning pattern comprise inference methods: modus ponens, modus tollens, syllogism; Proof methods, resolution theorem, Normal forms, and conversions between normal forms. Some of the deductions in propositional logic are shown in table 5.1.

5.2.1 Proof methods

There are two different methods, one is through *model checking* and other is *deduction* based. The first comprises enumeration of truth tables, and always always exponential in n, where n is size of the set of propositional symbols. The other, i.e., deduction based approach is repeated application of inference rules. The inference rules are used as operators in standard search algorithm. In fact, the application of inference approach to proof is called searching of solution. Proper selection of search directions are important here, as these will eliminate many unnecessary paths that are not likely to result to goal. Consequently, the proof based approach for reasoning is considered better and efficient compared to model enumeration/checking based method. The later is exhaustive and exponential in n, where n is size of the set of propositional symbols.

The property, logical system follows from the fundamental property of *monotonicity*. As per this, if $S \vdash \alpha$, and β is additional assertion, then $S \wedge \beta \vdash \alpha$.

Thereby application of inference rules is legitimate (*sound*) rule, which helps in generation of new knowledge from existing. If a search algorithm like DFS (depth first search) is used, it will always be possible to find the proof, as it will search the goal, what ever the depth may it be. Hence, the inference method in this case is *complete* also.

Before the inference rules are applied on the knowledge base, the existing sentences in the knowledge base (KB) needs to be converted into some *normal form.*

5.2.2 Resolution

The resolution is a inference using deduction. If two disjunctions have complementary literals, then a resultant inference of these is disjunction of these expressions, with complementary terms removed.

5.3 Inference Rules

The semantics of predicate logic provides a basis for formal theory of *logical inference*. It allows creation of new facts from the existing facts and rules.

An interpretation of a predicate statement means assignment of truth or false value to that statement. An interpretation that makes a sentence true is said to *satisfy* a sentence. An interpretation that satisfy every member of a set is said to satisfy the set.

An expression X *logically follows* from a set of expressions S, written as, $S \models X$, if every interpretation that satisfy S also satisfy X. That is, the knowledge-base S entails sentence x if and only if x is true in all *worlds* where knowledge-base is true.

The term logically follows simply means that x is true for every, potentially infinite interpretations that satisfy S. However, it it not a practical way of interpretations. In fact, inference rules provides a computationally feasible way to determine, when an expression logically follows.An inference rule is a mechanical process of producing new facts from the existing facts and rules.

An example of an inference rule is *Modus Ponens:*

$$\begin{array}{lcl} P & \Rightarrow & Q \\ P & & \\ \hline \therefore & Q & \end{array}$$

or $[(P \rightarrow Q) \wedge P] \rightarrow Q$, which is a valid statement or a tautology. Here, the Q also *logically follows* (entails) from $P \Rightarrow Q$ and P. That is, $[(P \Rightarrow Q) \wedge P] \models Q$. If a sentence x logically follows S, it is represented as $S \models x$. When every inference x from S logically follows S, then the inference system is *sound*. That is, $S \vdash x \Rightarrow S \models x$. If every x which logically follows S, can also be deducted (inferred), then the rule is *complete*. It is expressed as $S \models x \Rightarrow S \vdash x$. The sign '$\models$'is sign of 'logically follows'and '$\vdash$'is sign of 'deductions'. The *modus ponens* is sound and complete.

Another rule of inference is *modus tollens*, specified as:

$$\begin{array}{lcl} P & \Rightarrow & Q \\ \neg Q & & \\ \hline \therefore & \neg P & \end{array}$$

The reader may verify weather the inference rule of modus tollens is sound or complete or both or none?

References

[1] Chowdhary K.R. (2020) Logic and Reasoning Patterns. In: Fundamentals of Artificial Intelligence. Springer, New Delhi. https://doi.org/10.1007/978-81-322-3972-7_2

[2] Chowdhary, K.R., "Fundamentals of Discrete Mathematical Structures, 2nd Edition" *PHI India*, 2012, Chapter 4, 5.