32002: AI (First order Predicate Logic, Syntax and Semantics) **Spring 2014**

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7.1 Introduction

First-order logic is specifically designed for use as the basic theoretical instrument of a computer theorem proving program. From the theoretical point of view, however, an inference principle need only be *sound* (i.e., allow only logical consequences of premises to be deduced from them) and *effective* (i.e., it must be algorithmically decidable whether an alleged application of the inference principle is indeed an application of it).

The FOPL is more closer to natural language, and more powerful than propositional logic. It comprises variables, quantifiers, and functions.

7.2 Formal Preliminaries

Following are definitions of important related terminology.

Terms. A variable is a term, and a string of symbols consisting of a function symbol of degree $n \geq 0$ followed by n terms is a term.

Atomic formula. A string of symbols consisting of a predicate symbol of degree $n \geq 0$ followed by n terms is an atomic formula.

Literals. An atomic formula is a literal; and if A is an atomic formula then $\neg A$ is a literal.

Clauses. A finite set (possibly empty) of literals is called a clause. The empty clause is denoted by: []

Ground literals. A literal which contains no variables is called a ground literal.

Ground clauses. A clause, each member of which is a ground literal, is called a ground clause. In particular [] is a ground clause.

Well-formed expressions. Terms and literals are (the only) well formed expressions.

7.3 Predicate Logic

The first Order Predicate Logic(FOPL) offers formal approach to reasoning that has sound theoretical foundations. This aspect is important to mechanize the automated reasoning process where inferences should be correct and *logically sound*.

The statements of FOPL are flexible enough to permit the accurate representation of Natural languages. The words - *sentence* or *well formed formula* will be indicative of predicate statements. Following are some of the translations of English sentences into predicate logic:

- English sentence: Ram is man and Sita is women.

  Predicate form: $man(Ram) \vee woman(Sita)$

- English sentence: Ram is married to Sita.

  Predicate form: $married(Ram, Sita)$

- English sentence: Every person has a mother.

  Reorganized form of above: For all x, there exists a y, such that if x is person then x's mother is y.

  Predicate form: $\forall x \exists y[person(x) \Rightarrow hasmother(x, y)]$

- English sentence: If x and y are parents of a child z, and x is man, then y is not man.

  $\forall x \forall y[[isparents(x, z) \wedge isparents(y, z) \wedge man(x)] \Rightarrow \neg man(y)]$

We note that predicate language comprises constants: {Ram, Sita}, variables $\{x, y\}$, operators: $\{\Rightarrow, \wedge, \vee, \neg\}$, quantifiers: $\{\exists, \forall\}$, and functions/predicates: {married(x, y), person(x)}.

To signal that an expression is universally true, we use the symbol $\forall$, meaning 'for all'. Consider the sentence "any object that has a feathers is a bird."Its predicate is:

$$\forall_x[hasfeathers(x) \Rightarrow isbird(x)].$$

Then certainly,

$$hasfeathers(parrot) \Rightarrow isbird(parrot)$$

is true. Some expressions, although not always True, are True at least for some objects In logic, this is indicted by 'there exists', and the symbol used is $\exists$. For example, $\exists_x[bird(x)]$, when True, this expression means that there is at least one possible object, that when substituted in the position of x, makes the expression inside the brackets True.

Following are some examples of representations of knowledge FOPL.

Example 7.1 *Kinship Relations.*

Solution.

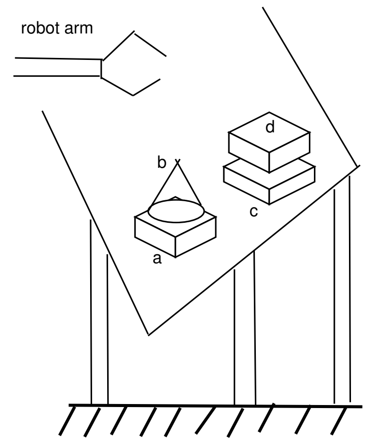

Figure 7.1: Blocks World.

$mother(priti, namrata)$ (That is, preeti's mother is namrata)

$mother(bharat, namrata)$

$father(priti, rajan)$

$father(bharat, rajan)$

$\forall x \forall y \forall z\ father(x, y) \vee mother(x, z) \Rightarrow areparent(y, z)$,

$\forall x \forall y \forall z\ mother(x, y) \wedge parent(z, y) \Rightarrow aresibling(x, z)$

In above, the predicate father(x, y) means x's father is y, and *areparents(x, y)* is a function. □

Example 7.2 *Blocks world. Consider that there are are physical objects, like -* cuboid, cone, cylinder *placed on the table-top, with some relative positions, as shown in figure 7.1. There are four blocks on the table:*a, c, d *are cuboid, and b is a cone. Along with these there is a robot arm, to lift one of the object having clear top.*

Solution. Following is the set of statements about the blocks world:

$cuboid(a)$

$cone(b)$

$cuboid(c)$

$cuboid(d)$

$onground(a)$

$onground(c)$

$ontop(b, a)$

$ontop(d, c)$

$toplcear(b)$

$topclear(d)$

$\exists x \exists y \; topclear(X) \wedge toplcear(Y) \wedge \neg cone(X) \Rightarrow puton(Y, X)$

The knowledge specified in the blocks world indicates that, objects a, c, d are cuboid, b is cone, a, c are put on the ground, and b, d are on a, c, respectively, and the top of b, d are clear. These are called facts in knowledge representation. At the end, the rule says that, there exists objects X and Y such that, both have their tops clear, and X is not a cone, then Y can be put on the object X. □

7.3.1 Interpretation of Predicate Expressions

A predicate statement is made of predicates, arguments(constants or variables), functions, operators, and quantifiers. Interpretation is process of assignment of truth values (True/False) to subexpressions, called atomic expressions, and computing the final value of any expression/statement. A statement or expressions in predicate logic is also called *wwf* (well formed formula).

Consider the interpretation of predicate formula:

$$\forall x[bird(x) \rightarrow flies(x)] \tag{7.1}$$

To find out the satisfiability of the formula 7.1, we need to substitute a value for x (an instance of x), and check if $flies(x)$ is true. Until that x is found, it may require instantiation with large number of values. Similarly, to check if 7.1 is valid, it may require infinitely large number of values in the domain of x to be verified, and if any one of that is false, the formula is not valid. Thus, checking of satisfiability as well as validity of a formula is predicate logic are complex. Given m number predicate sentence with each predicate having one argument having domain of size n, in the worst case it will require total n^m substitutions to test for satisfiability, as well as for validity checking. However, a sentence of m propositions will require in the worst case only 2^m substitutions. Hence, complexity of satisfiability checking in predicate sentences is much more complex than that in proposition logic.

It equally applies with expressions having existential quantifiers, like, $\exists x[bird(x) \rightarrow flies(x)]$

Thus, it is only the proof methods, using which logical deductions can be carried out in realistic times. For this the formulas must be converted to clause form (CNF), with only operators as $\wedge, \vee, \neg$, and not quantifiers.

Example 7.3 *Given, "All men are mortal"and "Socrates is man ", infer using predicate logic, that 'Socrates is mortal'.*

$$\begin{aligned} &[\forall_x \; man(x) \Rightarrow mortal(x)] \\ &man(socrates). \end{aligned} \tag{7.2}$$

and using the rule of "universal instantiation ", a variable can be instantiated by a constant, and universal quantifier can be dropped. Hence, from 7.2 we have

$$\begin{aligned} &man(socrates) \Rightarrow mortal(socrates) \\ &man(socrates). \end{aligned} \tag{7.3}$$

Using the rule of modus ponens on 7.3 we deduce $mortal(socrates)$. It is also *logical consequence*. If $\Gamma = [\forall_x man(x) \Rightarrow mortal(x)] \wedge man(socrates)$, and $\alpha = mortal(socrates)$, then we can say that $\Gamma \vdash \alpha$. □

References

[1] Chowdhary K.R. (2020) Logic and Reasoning Patterns. In: Fundamentals of Artificial Intelligence. Springer, New Delhi. `https://doi.org/10.1007/978-81-322-3972-7_3`

[2] Chowdhary, K.R., "Fundamentals of Discrete Mathematical Structures, 2nd Edition" *PHI India*, 2012, Chapter 6, 7.