32002: AI (FOPL-Conversion of predicate expression to clause Form) **Spring 2014**

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9.1 Clausal Form of formula

If $F_1, F_2, \ldots, F_n$ are wffs with $\wedge, \neg$ as operators in each of F_i, then $F_1 \vee F_2 \vee \cdots \vee F_n$ is called DNF (disjunctive normal form). Alternatively, if operators in F_i are $\vee, \neg$ then $F_1 \wedge F_2 \wedge \cdots \wedge F_n$ is called CNF (conjunctive normal form). The expression F_i is called a *term*. It is always possible to convert one normal form to another equivalent normal form.

However, we will prefer the CNF for predicate expression. Thus, for the purpose of reasoning to be carried out it is necessary to convert a predicate expression to CNF. For example, $\exists_x [p(x) \Rightarrow q(x)]$ can be converted to $\neg p(a) \vee q(a)$, where a is an instance of variable x. And, the expression $\neg p(a) \vee q(a)$ is a term of CNF. A formula in *CNF*, comprising $\wedge, \vee, \neg$ along with constants, variables, and predicates, is called *clausal* or *clause* form.

9.2 Conversion to Clausal Form

Following are the steps to convert a predicate formula into clausal-form.

1. Eliminate all the implications symbols using the logical equivalence: $p \rightarrow q \equiv \neg p \vee q$.
2. Move the outer negative symbol into the atom, for example, replace $\neg \forall_x p(x)$ by $\exists_x \neg p(x)$.
3. In an expression of nested quantifiers, existentially quantified variables not in the scope of universal quantifiers are replaced by constants. Replace $\exists x \forall y (f(x) \rightarrow f(y))$ by $\forall y (f(a) \rightarrow f(y))$.
4. Rename the variables if necessary.
5. Replace existentially quantified variables with *Skolem* functions; then eliminate corresponding quantifiers. For example, for $\forall_x \exists_y [\neg p(x) \vee q(y)]$, we obtain $\forall x [\neg p(x) \vee q(f(x))$. These newly created functions are celled *Skolem* functions, and the process is called *skolemization.*
6. Move the universal quantifiers to the left of that equation. For example, $\exists x [\neg p(x) \vee \forall y q(y)]$ to $\exists x \forall y [\neg p(x) \vee q(y)]$
7. Move the disjunctions down to the literals, i.e., terms should be connected by conjunctions only, vertically.
8. Eliminate the conjunctions.
9. Rename the variables, if necessary.

10. Drop all the universal quantifiers, and write each term in a separate line.

Example 9.1 *Convert the expression* $\exists x \forall y[\forall z P(f(x), y, z) \Rightarrow (\exists u Q(x, u) \wedge \exists v R(y, v))]$ *to clausal form.*

Solution. The steps discussed above are applied precisely, to get the clausal form of the predicate formula.

1. Eliminate implication.

$$\exists_x \forall_y [\neg \forall_z P(f(x), y, z) \vee (\exists_u Q(x, u) \wedge \exists_v R(y, v))]$$

2. Move negative symbol inside the atom.

$$\exists_x \forall_y [\exists_z \neg P(f(x), y, z) \vee (\exists_u Q(x, u) \wedge \exists_v R(y, v))]$$

3. Replace existentially quantified variables, not in the scope of universal quantifier, to constants.

$$\forall_y [\exists_z \neg P(f(a), y, z) \vee (\exists_u Q(a, u) \wedge \exists_v R(y, v))]$$

4. Rename variables (not required in this example.)

5. Replace existentially quantified variables that are functions of universal quantified variables, by skolem functions:

$$\forall_y [\neg P(f(a), y, g(y) \vee (Q(a, h(y) \wedge R(y, l(y))]$$

6. Not required in this example.

7. Move disjunctions down to literals.

$$\forall_y [(\neg P(f(a), y, g(y)) \vee (Q(a, h(y))) \wedge (\neg P(f(a), y, g(y)) \vee R(y, l(y)))]$$

8. Eliminate conjunctions.

$$\forall_y [\neg P(f(a), y, g(y)) \vee (Q(a, h(y)), (\neg P(f(a), y, g(y)) \vee R(y, l(y))]$$

9. Not required in this example.

10. Drop all universal quantifiers and write each term on separate line.

$$\neg P(f(a), y, g(y)) \vee (Q(a, h(y)),$$
$$\neg P(f(a), y, g(y)) \vee R(y, l(y)).\square$$

References

[1] Chowdhary K.R. (2020) Logic and Reasoning Patterns. In: Fundamentals of Artificial Intelligence. Springer, New Delhi. `https://doi.org/10.1007/978-81-322-3972-7_3`