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3.1 Properties of Inference Rules

An inference rule is a mechanical process of producing new facts from the existing facts and rules. The semantics of predicate logic provides a basis for formal theory of *logical inference*. It allows creation of new facts from the existing facts and rules.

An interpretation of a predicate statement means assignment of truth or false value to that statement. An interpretation that makes a sentence true is said to *satisfy* a sentence. An interpretation that satisfy every member of a set is said to satisfy the set.

Definition 3.1 *Logically follows.*

If every interpretation that satisfy S also satisfy X, then we say expression X *logically follows* from a set of expressions S (the *knowledge base*). In other words, the knowledge-base S entails sentence X if and only if X is true in all *worlds* where knowledge-base is true. If a sentence X logically follows S, we represent it as $S \models X$. □

The term logically follows simply means that X is true for every, potentially infinite interpretations that satisfy S. However, it it not a practical way of interpretations. In fact, inference rules provides a computationally feasible way to determine, when an expression logically follows.

An example of an inference rule is *Modus Ponens:*

$$[(P \Rightarrow Q) \wedge P] \Rightarrow Q \tag{3.1}$$

which is a valid statement (a tautology). Here, the Q also *logically follows* (entails) from $(P \Rightarrow Q) \wedge P$. That is, $[(P \Rightarrow Q) \wedge P] \models Q$.

Definition 3.2 *'Sound'inference system.*

When every inference X deduced from S also logically follows S, then the inference system is *sound*. This is expressed by,

$$S \vdash x \Rightarrow S \models x. \tag{3.2}$$

The '$\vdash$'is sign of 'deduction'.

Soundness means that you cannot prove anything that is wrong.

□

Definition 3.3 *'Complete'inference system.*

If every X which logically follows S can also be deducted (inferred), then the inference rule is *complete*. This is expressed by

$$S \models x \Rightarrow S \vdash x. \tag{3.3}$$

Completeness means that you can prove anything that is right.

□

Another rule of inference is *Modus tollens*, specified as,

$$[(P \Rightarrow Q) \wedge \neg Q] \Rightarrow \neg P \tag{3.4}$$

is sound and complete.

The reader may verify weather the inference rule of modus tollens is sound or complete or both or none?

3.2 Nonmonotonic Reasoning

Nonmonotonic logic is the study of those ways of inferring additional information from given information that do not satisfy the *monotonicity* property satisfied by all methods based on classical logic. In Mathematics, if a conclusion is warranted on the basis of certain premises, no additional premises will ever invalidate the conclusion.

In everyday life, however, it seems clear that we, human beings, draw sensible conclusions from what we know and that, on the face of new information we often have to take back previous conclusions, even when the new information we gathered in no way made us want to take back our previous assumptions (see figure 3.1).

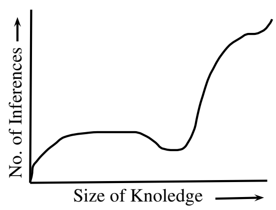

Figure 3.1: Nonmonotonic Reasoning.

For example, we may hold the assumption that most birds fly, but that penguins are birds that do not fly and, learning that Tweety is a bird, infer that it flies. Learning that Tweety is a penguin, will in no way make us change our mind about the fact that most birds fly and that penguins are birds that do not fly, or about the fact that Tweety is a bird. It should make us abandon our conclusion about its (tweety's)flying capabilities, though. It is most probable that intelligent automated systems will have to do the same kind of (nonmonotonic) inferences.

Considering that S is set of sentences of propositional logic, and α is inferred from it, i.e $S \vdash \alpha$. For any new propositional sentences β, if $\beta \cup S \vdash \alpha$ then it is *monotonic* reasoning. If it is not necessary that $S \cup \beta \vdash \alpha$, then it is called *nonmonotonic* reasoning.

Some of the systems that perform such nonmonotonic inferences are - *negation as failure*, *circumscription*, *modal system*, *default logic*, *autoepistemic logic*, and *inheritance systems*.